

Equivalence Properties by Typing in Cryptographic Branching Protocols

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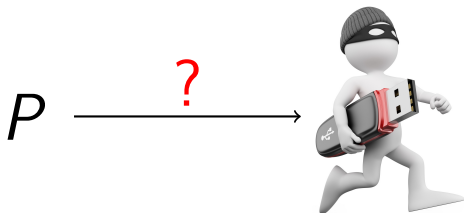
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Trace properties

Trace properties = satisfied by all traces of a protocol

Example: reachability properties:

Can the attacker learn a given message ?

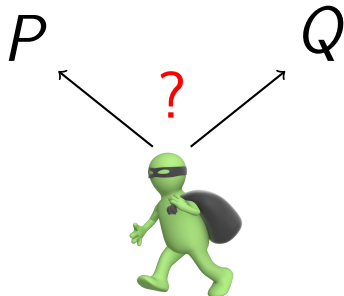


⇒ secrecy, authentication, ...

Equivalence

Some properties require the notion of **equivalence**:

Are two protocols indistinguishable for an attacker?

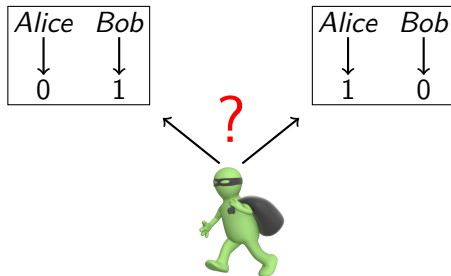


Example:

vote privacy, strong flavours of secrecy, anonymity, unlinkability, ...

Example: vote privacy

Example: Privacy of the vote in voting protocols



Alice and Bob vote for either 0 or 1.

The values of the votes = 0 and 1 are **not** secret

The votes are secret if:

$$Alice(0) \mid Bob(1) \approx Alice(1) \mid Bob(0)$$

Type systems

Idea: design a type system that ensures protocols satisfy security properties

- Type systems: already applied to trace properties

$$M : Secret \vdash P \implies M \text{ is not deducible in } P$$

- Now: for equivalence

$$\vdash P \sim Q \implies P \approx Q$$

- Efficient (though incomplete) procedures
- Modularity

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Problem:

- Usually: typing \rightarrow **overapproximate** the set of traces.
- Sound for trace properties, but **not equivalence**
 \rightarrow might miss that some traces are only possible for P and not Q

- **Step 1:** $\vdash P \sim Q : C$
typing to ensure **no leaks in behaviours**
collect all symbolic messages sent on the network into a *constraint*
- **Step 2:** $check(C)$
ensure there are **no leaks in the messages sent**
→ checking for repetitions

Example:

$$C = \{\text{enc}(x, k) \sim \text{enc}(a, k), \text{enc}(y, k) \sim \text{enc}(b, k)\}$$

If in some execution we can have $x = y$, equivalence is broken.

Main result: Soundness

Theorem (Soundness)

If $\Gamma \vdash P \sim Q : C$ and $\forall \theta. C\theta$ does not leak information, then

$$P \approx Q$$

Theorem (Procedure to check constraints)

$check(C) \Rightarrow \forall \theta. C\theta$ does not leak information.

Hypotheses:

- atomic keys only
- fixed cryptographic primitives: symmetric and asymmetric encryption, signature, hash, pairing
- no replication (bounded number of sessions only)

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From two to unbounded number of sessions

If one session typechecks, then any number of sessions typecheck:

Theorem (informal)

$$\Gamma \vdash P \sim Q : C \implies \Gamma \vdash !P \sim !Q : !C$$

How to check that $!C$ does not leak information?

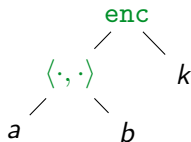
→ It is sufficient to check two copies of C :

Theorem (informal)

$$\text{check}(C \cup C') \implies \text{check}(!C)$$

Symbolic model

- Messages are **terms** constructed using **abstract cryptographic primitives**,



- **Symbolic attacker** with abilities defined by **deduction rules**

$$\frac{\text{enc}(x, y) \quad y}{x} \qquad \frac{x \quad y}{\langle x, y \rangle}$$

Process algebra similar to the applied pi-calculus

$$\begin{array}{l} P, Q ::= \\ \quad 0 \\ \quad | \text{ new } n.P \\ \quad | \text{ out}(M).P \\ \quad | \text{ in}(x).P \\ \quad | P \mid Q \\ \quad | \text{ let } x = d(y) \text{ in } P \text{ else } Q \\ \quad | \text{ if } M = N \text{ then } P \text{ else } Q \\ \quad | !P \end{array}$$

Static equivalence

Frames are sequences of messages modelling the attacker's knowledge

$$\phi = \{x_1 \mapsto k, x_2 \mapsto a, x_3 \mapsto \text{enc}(b, k)\}$$

Static equivalence = indistinguishability of frames

$$\phi \approx \phi' \iff \forall R, S. R\phi = S\phi \Leftrightarrow R\phi' = S\phi'$$

Example:

$$\{\text{enc}(a, k)\} \approx \{\text{enc}(b, k)\}$$

but

$$\{\text{enc}(a, k), \text{enc}(a, k)\} \not\approx \{\text{enc}(a, k), \text{enc}(b, k)\}$$

and

$$\{k, \text{enc}(a, k)\} \not\approx \{k, \text{enc}(b, k)\}$$

Trace equivalence

A **trace** (tr, ϕ) is a sequence of observable actions
+ a frame of messages sent on the network

Definition (Trace equivalence)

P and Q are trace equivalent if any trace of P
can be mimicked by a trace of Q (and conversely)

i.e.

$$\forall (tr, \phi) \in \text{trace}(P). \exists (tr, \phi') \in \text{trace}(Q). \phi \approx \phi'$$

and

$$\forall (tr, \phi) \in \text{trace}(Q). \exists (tr, \phi') \in \text{trace}(P). \phi \approx \phi'$$

Typing messages

Types for messages :

$$\begin{array}{l} l ::= LL \mid HL \mid HH \\ T ::= l \\ \quad | \text{key}^l(T) \\ \quad | T * T \\ \quad | T \vee T \\ \quad | \dots \end{array}$$

- labels = levels of **confidentiality** and **integrity**
 - **LL** for public messages
 - **HH** for secret values
- key types $\text{key}^l(T)$

Example:

$$\text{key}^{\text{HH}}(\text{LL} * \text{HH})$$

Typing messages

$$\frac{\Gamma \vdash M \sim N : T \quad \Gamma(k) = \text{key}^{\text{HH}}(T)}{\Gamma \vdash \text{enc}(M, k) \sim \text{enc}(N, k) : \text{LL}}$$

Ensure the messages sent are **safe to output**:

→ **similar structure**

$$\langle a, b \rangle \not\sim a$$

$$\text{enc}(\langle a, b \rangle, k) \sim \text{enc}(a, k) \quad \text{only if } k \text{ is secret}$$

$$\frac{\Gamma \vdash M \sim N : T \rightarrow c \quad \Gamma(k) = \text{key}^{\text{HH}}(T)}{\Gamma \vdash \text{enc}(M, k) \sim \text{enc}(N, k) : \text{LL} \rightarrow c \cup \{\text{enc}(M, k) \sim \text{enc}(N, k)\}}$$

- Establish **invariants** regarding the types of keys
If k is **secret**, the **type of M, N** must match the **type of k**
- Collect **constraints**
Here we add the couple $\text{enc}(M, k) \sim \text{enc}(N, k)$ to the **constraint**

Typing processes

- All **output messages** must be of type **LL**
- Their **constraints** are collected

$$\frac{\Gamma \vdash M \sim N : \mathbf{LL} \rightarrow c \quad \Gamma \vdash P \sim Q : \mathbf{C}}{\Gamma \vdash \text{out}(M).P \sim \text{out}(N).Q : \mathbf{C} \cup c}$$

- All **input messages** are considered to be of type **LL**:

$$\frac{\Gamma, x : \mathbf{LL} \vdash P \sim Q : \mathbf{C}}{\Gamma \vdash \text{in}(x).P \sim \text{in}(x).Q : \mathbf{C}}$$

Typing processes (2)

→ Processes have to progress the same way:

accept **inputs/outputs** at the same time,
follow (typably) equivalent branches

Example: applying destructors

$$\frac{\Gamma(x) = \text{LL} \quad \Gamma(k) = \text{key}^{\text{HH}}(T) \quad \Gamma, y : T \vdash P \sim Q : C \quad \Gamma \vdash P' \sim Q' : C'}{\Gamma \vdash \text{let } y = \text{dec}(x, k) \text{ in } P \text{ else } P' \sim \text{let } y = \text{dec}(x, k) \text{ in } Q \text{ else } Q' : C \cup C'}$$

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Example: If k is a secret key

$\text{out}(\text{enc}(a, k)) \sim \text{out}(\text{enc}(b, k))$ is fine

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but not both together

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$$\mathcal{C} = \{\text{enc}(a, k) \sim \text{enc}(b, k), \text{enc}(a, k) \sim \text{enc}(a, k)\}$$

Collect symbolic messages in a **constraint** C while typing
and check that it is **consistent**

i.e. for any possible instantiation, C instantiated does not leak anything:

$$C = \{u_1 \sim v_1, \dots, u_n \sim v_n\}$$

must satisfy

$$\forall \theta, \theta'. \quad \{u_1\theta, \dots, u_n\theta\} \approx \{v_1\theta', \dots, v_n\theta'\}$$

Constraints: Checking consistency

- **Open messages** as much as possible:

$$\begin{aligned}\langle M, N \rangle &\longrightarrow M, N \\ \text{enc}(M, k) &\longrightarrow M \quad \text{if } k \text{ has type } \text{key}^{\text{LL}}(\cdot) \\ &\dots\end{aligned}$$

- Check that both sides of the opened constraint satisfy the **same equalities** once instantiated (unification)

$$M \sim N, M' \sim N' \in C$$

$$\forall \theta, \theta'. M\theta = M'\theta \iff N\theta = N'\theta'$$

- Actually only consider **well-typed** θ, θ'
i.e.

$$\forall x. \vdash \theta(x) \sim \theta'(x) : \Gamma(x)$$

The case of different keys

In the rules shown before, the keys were **the same** on both sides

$$\frac{\Gamma \vdash M \sim N : T \rightarrow c \quad \Gamma(k) = \text{key}^{\text{HH}}(T)}{\Gamma \vdash \text{enc}(M, k) \sim \text{enc}(N, k) : \text{LL} \rightarrow c \cup \{\text{enc}(M, k) \sim \text{enc}(N, k)\}}$$

→ How to handle more complex cases where **different keys** are used?

Example: anonymity, unlinkability

Example: Private Authentication

→ Authenticating B to A **anonymously** to others

$$A \rightarrow B : \text{aenc}(\langle N_a, \text{pk}(k_a) \rangle, \text{pk}(k_b))$$

$$B \rightarrow A : \begin{cases} \text{aenc}(\langle N_a, \langle N_b, \text{pk}(k_b) \rangle \rangle, \text{pk}(k_a)) & \text{if } B \text{ accepts } A\text{'s request} \\ \text{aenc}(N_b, \text{pk}(k)) & \text{if } B \text{ declines } A\text{'s request} \end{cases}$$

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Anonymity: an attacker cannot learn whether B is willing to talk to A or not

$$\text{Alice} \mid \text{Bob}(\text{pk}_{\text{Alice}}) \approx \text{Alice} \mid \text{Bob}(\text{pk}_{\text{Charlie}})$$

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Problems: different keys and non uniform branching

→ We introduce **bikeys**: pairs of keys with a type

Example:

$$\Gamma(k_1, k_2) = \text{key}^{\text{HH}}(\text{LL} * \text{HH})$$

There may be **multiple bindings** for the same key :

$$\Gamma(k_1, k_2) = \text{key}^{\text{HH}}(\text{LL} * \text{HH})$$

$$\Gamma(k_1, k_3) = \text{key}^{\text{HH}}(\text{HH} * \text{LL})$$

We also add a type specifying that the keys are actually **the same**:

$$\Gamma(k, k) = \text{eqkey}^{\text{HH}}(\text{HH})$$

Bikeys: encrypting

→ The rules for encrypting go as expected:
allow any pair of keys that is **valid in Γ**

$$\frac{\Gamma \vdash M \sim N : T \rightarrow c \quad \Gamma(k_1, k_2) = \text{key}^{\text{HH}}(T)}{\Gamma \vdash \text{enc}(M, k_1) \sim \text{enc}(N, k_2) : \text{LL} \rightarrow c \cup \{\text{enc}(M, k_1) \sim \text{enc}(N, k_2)\}}$$

→ Similarly for asymmetric encryption and signature

Previously:

$$\frac{\Gamma(x) = \text{LL} \quad \Gamma(k) = \text{key}^{\text{HH}}(T) \quad \Gamma, x : T \vdash P \sim Q : C \quad \Gamma \vdash P' \sim Q' : C'}{\Gamma \vdash \text{let } y = \text{dec}(x, k) \text{ in } P \text{ else } P' \sim \text{let } y = \text{dec}(x, k) \text{ in } Q \text{ else } Q' : C \cup C'}$$

With different keys ?

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Problem: There may be several bindings for k_1 in Γ
 x may be encrypted with k_1 on the left, $k_3 \neq k_2$ on the right

→ We do not know that decryption succeeds or fails equally

= the processes may branch **non uniformly** i.e. follow different branches

The problem of non-uniform branching

How to handle cases where the processes follow different branches?

- when decrypting with bikeys
- conditional branching where uniform execution cannot be ensured

→ We have to take **all cases** into account:

$$\frac{\begin{array}{l} \Gamma(y) = \text{LL} \quad \Gamma(k_1, k_2) = \text{key}^{\text{HH}}(T) \\ \Gamma, x : T \vdash P \sim Q \rightarrow C \quad \Gamma \vdash P' \sim Q' \rightarrow C' \\ (\forall T'. \forall k_3 \neq k_2. \Gamma(k_1, k_3) = \text{key}^{\text{HH}}(T') \Rightarrow \Gamma, x : T' \vdash P \sim Q' \rightarrow C_{k_3}) \\ (\forall T'. \forall k_3 \neq k_1. \Gamma(k_3, k_2) = \text{key}^{\text{HH}}(T') \Rightarrow \Gamma, x : T' \vdash P' \sim Q \rightarrow C'_{k_3}) \end{array}}{\Gamma \vdash \text{let } x = \text{dec}(y, k_1) \text{ in } P \text{ else } P' \sim \text{let } x = \text{dec}(y, k_2) \text{ in } Q \text{ else } Q' \rightarrow C \cup C' \cup \left(\bigcup_{k_3} C_{k_3}\right) \cup \left(\bigcup_{k_3} C'_{k_3}\right)}$$

Note: the simple rule still applies when keys have type $\text{eqkey}'(T)$

Back to Private Authentication

$A \rightarrow B : \text{aenc}(\langle N_a, \text{pk}(k_a) \rangle, \text{pk}(k_b))$

$B \rightarrow A : \begin{cases} \text{aenc}(\langle N_a, \langle N_b, \text{pk}(k_b) \rangle \rangle, \text{pk}(k_a)) & \text{if } B \text{ accepts } A\text{'s request} \\ \text{aenc}(N_b, \text{pk}(k)) & \text{if } B \text{ declines } A\text{'s request} \end{cases}$

$Alice \mid Bob(\text{pk}_a) \approx Alice \mid Bob(\text{pk}_c)$

We can typecheck Bob's response by having bindings in Γ for all cases

- (k_a, k) authentication succeeds on the left, fails on the right
- (k, k_c) authentication succeeds on the right, fails on the left
- (k_a, k_c) authentication succeeds on both sides
- (k, k) authentication fails on both sides

Done?

Done? Not. Yet.

The case of dynamic keys

In the rules shown before, the keys were all **fixed, long-term keys**

→ We also want to consider **key distribution** mechanisms, where keys are

- **generated** (session keys)
- **received** from the network and then used to encrypt, decrypt, sign

The case of dynamic keys (2)

→ A new type for **session keys**

$\text{seskey}'(T)$

Processes can

- **generate** session keys (must specify a **type annotation**)

$$\frac{\Gamma, (k, k) : \text{seskey}'(T) \vdash P \sim Q : C}{\Gamma \vdash \text{new } k : \text{seskey}'(T). P \sim \text{new } k : \text{seskey}'(T). Q : C}$$

- **receive** and store session keys in variables of type $\text{seskey}'(T)$
- use these **variables as keys** to encrypt, decrypt, ...

The case of dynamic keys (3)

→ Tricky point: consistency of the constraints

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Example: If $x : LL$ (key provided by the attacker), typechecking

$$\text{out}(\text{enc}(a, x)) \sim \text{out}(\text{enc}(a, x))$$

yields the constraint

$$\text{enc}(a, x) \sim \text{enc}(a, x)$$

The case of dynamic keys (3)

→ Tricky point: **consistency of the constraints**

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yields the constraint

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If we proceed as before and **open the messages** we get

$$x \sim x$$

which typically renders the constraint **inconsistent**

The case of dynamic keys (4)

Indeed: as soon as C contains

$$x \sim x \text{ and } M \sim N$$

if we choose $\theta(x) = M$ and $\theta'(x) \neq N$,

C instantiated with θ, θ' is **not statically equivalent**

The case of dynamic keys (4)

Indeed: as soon as C contains

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if we choose $\theta(x)=M$ and $\theta'(x) \neq N$,

C instantiated with θ, θ' is **not statically equivalent**

→ We need to further **restrict** the θ we consider

The case of dynamic keys (4)

Indeed: as soon as C contains

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if we choose $\theta(x)=M$ and $\theta'(x) \neq N$,

C instantiated with θ, θ' is **not statically equivalent**

→ We need to further **restrict** the θ we consider

→ **Invariant**: variables of type **LL** only contain messages
the attacker can **construct** from the remainder of the constraint

The case of dynamic keys (4)

Indeed: as soon as C contains

$$x \sim x \text{ and } M \sim N$$

if we choose $\theta(x)=M$ and $\theta'(x) \neq N$,

C instantiated with θ, θ' is **not statically equivalent**

→ We need to further **restrict** the θ we consider

→ **Invariant**: variables of type **LL** only contain messages
the attacker can **construct** from the remainder of the constraint

→ Prevents the previous θ, θ' and solves the problem

Experimental results

- Prototype implementation for our type system
- We implement a typechecker, together with the procedure for constraints
- Very efficient
- But requires some type annotations

Protocol	Akiss	Apte	Apte-POR	Spec	Sat-Eq	TypeEq
Denning-Sacco	10	6	12	7	>30	>30
Wide Mouth Frog	14	7	12	7	>30	>30
Needham-Schroeder Symmetric Key	10	6	10	6	>30	>30
Yahalom-Lowe	10	6	10	7	>30	>30
Otway-Rees	6	3	6	6	-	>30
Needham-Schroeder-Lowe	8	4	4	4	-	>20

Number of sessions treated when proving secrecy
(bounded case)

Closer look for the Needham-Schroeder symmetric key protocol:

# sessions	Akiss	Apte	Apte-POR	Spec	Sat-Eq	TypeEq
3	0.1s	0.4s	0.02s	52s	0.2s	0.003s
6	20s	TO	4s	MO	0.4s	0.003s
7	2m		8m		1.3s	0.003s
10	SO		TO		2.3s	0.005s
12					4s	0.005s
14					7s	0.007s
30					1m6s	0.01s

Experimental results (unbounded)

We also compare to ProVerif for unbounded numbers of sessions:

Protocols	ProVerif	TypeEq
Helios	x	0.005s
Needham-Schroeder (sym)	0.23s	0.016s
Needham-Schroeder-Lowe	0.08s	0.008s
Yahalom-Lowe	0.48s	0.020s
Private Authentication	0.034s	0.008s
BAC	0.038s	0.005s

- Performances comparable to ProVerif for unbounded numbers of sessions
- First automated proof for Helios with unbounded number of sessions without private channels

Conclusion and future work

- a new approach to automatic proofs of equivalence properties for cryptographic protocols
- based on type systems + constraints
- handle bounded and unbounded number of sessions (CCS'17), dynamic keys, bikeys and non uniform branching (POST'18)
- efficient implementation

Future work:

- type inference
- computational soundness
- composition